The kinematics of a bouncing ball

An inflated plastic ball bounces on a tiled floor. A hand-held Vernier motion detector records times of flight and computer calculations give the position-time and velocity-time graphs below.

If we can neglect air resistance the acceleration of the ball will be constant when the ball is clear of the floor. Velocity-time lines on the lower graph will be straight (as shown) with a slope close to the acceleration due to gravity, -9.8 m/s/s.
When the ball is in contact with the floor it’s acceleration is vertical and rises to a maximum at maximum compression. The time on the floor at each bounce is the same if the ball does half a period of simple harmonic motion. The graphs below appear to confirm these expectations.

![Graph of ball acceleration](image1)

Subtracting the constant acceleration due to gravity and calculating the force as $f = ma$ with a measured mass of 61 grams for the ball gives the graph below that shows the force on the floor as the ball bounces several times and then comes to rest.

![Graph of force on floor](image2)

Dividing the integral in Ns by the time (4 seconds) gives (within errors) the average force on the floor as the weight of the ball. ($2.46/4.0 = 0.61$ N).
The average force on the floor

Suppose a ball bounces on a vertical line without loss of energy, returning to the same height after each rebound. The impulse delivered to the floor by each impact will be the change in momentum \( \Delta p \).

\[
\Delta p = 2mu
\]

…. where \( u \) is the initial (maximum) vertical velocity.

The velocity at maximum height is zero and the constant acceleration is \(-g\). Using \( v = u + at \), the time of flight \( \Delta t \) (between impacts) is \( 2u/g \).

The rate of change of momentum per second is given by …

\[
\frac{\Delta p}{\Delta t} = \frac{2mu}{2u/g} = mg
\]

This rate of change of momentum gives the average force on the floor, independent of the rebound velocity \( u \) and consequently the bounce height.

In a real case bounce height reduces after each impact because some kinetic energy is converted to heat, but the average force on the floor will remain equal to the weight of the ball, as confirmed by the demonstration, since \( \Delta p/\Delta t \) is independent of bounce height.

Notes

The graphs are first approximations. Air resistance will make the acceleration on the way up greater than, and on the way down less than, 9.8. The slopes of the segments here are very close to the same. To get a more accurate value for the acceleration due to gravity check the temperature setting in Logger Pro, use a heavier ball, drop it from a greater height, and consider reducing the default value for the derivative calculation from averaging over seven points to three.

The actual peak forces are not accurately shown in the last two graphs. The values are functions of the number of points used in the derivation of the acceleration from time of flight data. The same is true for the times in contact with the floor. The indicated times are about the same, but are longer than the actual times due to calculation errors.

The weight of the ball being the integral of the force-time graph is a matter of kinematics, the definitions of variables. Agreement is a matter of checking the numerical calculations within the computer, not the physics of the situation.