George Gabriel Stokes was an Irish-born mathematician who is most famous for his work describing the motion of a sphere through a viscous fluid. His equation describes the force needed to move a small sphere through a continuous, quiescent fluid at low velocity (without turbulence).

\[ f = 6\pi \eta rv \]

... where \( r \) is the radius of the sphere, \( v \) is the terminal velocity and \( \eta \) (eta) is the viscosity of the fluid.

If the drag force \( f \) on a falling sphere of radius \( r \) and velocity \( v \) are known, \( \eta \) can be found at a particular temperature.

The term viscosity has no close equivalent in English. The meaning is best explained with an example. For instance: honey is more viscous than water and warming honey makes it easier to pour as it becomes less viscous. The measurement of \( \eta \) from first principles for peanut butter is demonstrated here …[pdf]. Please read this document for the definition of \( \eta \) as shearing stress over velocity gradient and a discussion of units.

The viscosities of a selection of common fluids are listed in the table below.

<table>
<thead>
<tr>
<th>Substance</th>
<th>Temp. °C</th>
<th>Viscosity Ns/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>Room temp</td>
<td>1.35x10⁻⁵</td>
</tr>
<tr>
<td>Water</td>
<td>0 °C</td>
<td>1.79x10⁻³</td>
</tr>
<tr>
<td></td>
<td>37 °C</td>
<td>0.82x10⁻³</td>
</tr>
<tr>
<td></td>
<td>100 °C</td>
<td>0.29x10⁻³</td>
</tr>
<tr>
<td>Glycerin</td>
<td>0 °C</td>
<td>1.200x10⁻³</td>
</tr>
<tr>
<td></td>
<td>30 °C</td>
<td>0.070x10⁻³</td>
</tr>
<tr>
<td>Castor oil</td>
<td>37 °C</td>
<td>2.97x10⁻³</td>
</tr>
<tr>
<td>Mercury</td>
<td>37 °C</td>
<td>1.46x10⁻³</td>
</tr>
<tr>
<td>Ethanol</td>
<td>37 °C</td>
<td>0.99x10⁻³</td>
</tr>
<tr>
<td>Methanol</td>
<td>37 °C</td>
<td>0.47x10⁻³</td>
</tr>
<tr>
<td>Goutien syrup</td>
<td>12 °C</td>
<td>1.40 × 10⁻³</td>
</tr>
<tr>
<td>Liquid air</td>
<td>-152 °C</td>
<td>0.17x10⁻⁵</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>37 °C</td>
<td>1.57x10⁻⁵</td>
</tr>
<tr>
<td>Hydrogen gas</td>
<td>37 °C</td>
<td>0.91x10⁻⁵</td>
</tr>
</tbody>
</table>
In the table above note the dependence of viscosity on temperature for water and for glycerin, which is often used in demonstrations and student labs. Note the relative viscosity of ethanol and water at 37°C.

**Wall and end effects**

If paper muffin cups are dropped in clear acrylic tubes it immediately becomes obvious that the terminal velocity is reduced by the proximity of the walls.

**The wall effect**

The *wall effect* correction introduced in 1920 by Gibson and Jacobs accounts for the increased drag against the sides of the container holding the fluid as the sphere descends and is based on the ratio of the sphere radius ($r$) to the inner radius of the cylinder ($R$). The first order correction to find $v_T$ (the unimpeded terminal velocity) with no higher powers of ($r/R$) is ...

$$v_T = v(1 + 2.4 \frac{r}{R})$$ … where $v$ is the measured terminal velocity.

This correction neglects higher powers of $r/R$.

**The end effect**

The *end effect* correction, which modifies Stokes' law to account for the fact that the sphere does not fall indefinitely, is based on the ratio of sphere radius to the total height of the liquid. The effect of this correction is usually much smaller than the effect of the wall correction and can often be neglected.

**Possible investigations**

*If glass beads are dropped in a liquid like glycerin or golden syrup the wall effect can be studied in measuring cylinders - provided the velocity of descent is low enough to maintain laminar flow. (Look up Reynolds number on the web.)*

*Wall effects for turbulent flow could be studied in water and alcohol in measuring cylinders, or in air with paper muffin cups in clear acrylic tubes. Video analysis would be a suitable way to collect data.*

A literature search will return studies that extend the early work in this field, including the characteristics of fall in non-newtonian liquids.

For a biography of Stokes see ...

http://www-groups.dcs.standrews.ac.uk/~history/Biographies/Stokes.html
Stokes law and laminar flow

In this demonstration spheres of equal radius and different densities fall in honey or golden syrup. The difficulty is obtaining small spheres of equal radius and densities not too far above that of the liquid chosen. That problem is solved by using plastic BB’s that come in a range of densities with closely controlled radii.

Fig 1 – BB’s are sold in different weights. For this demonstration cheap examples were collected from different sources and the mass of each type was determined by weighing on a sensitive balance.

The time of descent between two levels marked with rubber bands (to avoid large parallax errors) in a tall jar of honey close to the centre line (to avoid the edge effect) was measured with a stopwatch.

Fig 2 – A tall jar of honey

The difficulty with using Stokes law to determine the viscosity of a liquid in a student laboratory lies in ensuring that the sphere (ball bearing) falls in laminar flow. The writer remembers doing this exercise himself with engine oil and a small steel ball bearing when he was himself at school in M5 (year 11). There was no independent confirmation of the value obtained, which must have been in error because of the high terminal velocity that generated turbulent flow.
To avoid the complication of turbulent flow and return a realistic value spheres of the same radius and different mass are required. If terminal velocity is found to be proportional to the mass a constant radius Stokes law is followed and the value of \( \eta \) will be reliable.

Data for BB’s in honey are plotted below.

![Graph 1](image)

**Graph 1** – velocity versus mass for plastic BB’s descending at terminal velocity in honey.

For laminar flow …

\[
(M-m)g = (6\pi \eta r)v_T
\]

… where \( M \) is the mass of the sphere, \( m \) is the mass of the equivalent volume of honey, and \( v_T \) is the terminal velocity.

The linear fit shows that within errors the flow is laminar since terminal velocity is proportional to \( (M-m) \), (the mass allowing for the buoyancy force due to the honey found from the intercept on the mass axis).

*It is left as an exercise to find \( \eta \), the coefficient of viscosity for this particular sample of honey at 24°C. The radius of the balls was 2.15 mm.*

Acknowledgement

Thanks are due to Jon Eales at ISB for providing an initial set of BB’s for these measurements.