Coefficients of Restitution

Introduction

The coefficient of restitution for a two-body collision is defined as the ratio of relative velocities after impact to relative velocities on approach. When a ball bounces vertically on a level floor (the earth) the ratio simplifies to the rebound velocity over the impact velocity. Since KE is converted to PE as a body rises, when air resistance can be neglected the coefficient of restitution is the square root of the ratio of maximum heights.

\[ C_r = \left( \frac{\Delta h_{n+1}}{\Delta h_n} \right)^{0.5} \] …… [1]

Rubber balls

Energy converted to heat in a rubber ball when it bounces on tiles or concrete is mainly due to hysteresis in the rubber [pdf]. Hysteresis loops on a force compression graph increase greatly in area with the maximum compression of the ball and the coefficient of restitution is not expected to be independent of the impact energy. That this is the case for a basketball dropped from an initial height of 9 metres can be seen in figure 1.

![Graph showing coefficient of restitution vs. bounce height](image)

**Fig 1** – approximate coefficients of restitution for a standard basket-ball bouncing on concrete.

**Note**: the coefficients in figure 1 above were obtained from relative heights. Since air resistance may have been significant, in further work, relative velocities before and after impact should be found from video analysis or otherwise.
The resilience of rubber and most similar polymers increases with rising temperature in the zero to 100 °C range. Most common rubber balls, solid or inflated, have increased coefficients of restitution at increased temperatures. Measurements at low temperature can be made without a controlled environment by submerging the ball in a mixture of dry ice and alcohol, using tongs to lift it out and immediately dropping it. Video analysis, motion detection, or sound recording (see below) can be used to find the coefficient of restitution.

Note: if liquid nitrogen is used the polymer is likely to pass below the glass transition temperature and may shatter.

Fig 2 – typical coefficients of restitution as a function of temperature for a solid rubber ball. Data: Yoshitaka Tamiya, ISB Journal of Physics, Vol, 4.

Squash balls

Squash balls are slammed repeatedly against a wall before a game to warm them and increase the bounce. The increased coefficient of restitution for warm squash balls is said to be due mainly to the change in the properties of the rubber, not to increased pressure inside the ball.

Anomalous balls

Pairs of solid balls of apparently similar black rubber are sold as “sad and happy balls”. The happy ball is a normal neoprene rubber bouncy type, and the sad ball is made from polynorborene (a polymer that has high hysteresis). Polynorbornene balls have a low coefficient of restitution at room temperature that increases as the temperature is lowered. Table-tennis balls behave in a similar way. The coefficient of restitution increases as the temperature is lowered.
Demonstrations

1 A modified table-tennis ball

A small hole is put in a 2.60 g table-tennis ball. The ball is allowed to bounce on tiles below a motion detector.

Fig 3 – a position-time plot for a bouncing table-tennis ball.

Increasing amounts of loose tissue paper are added to the ball. The maximum heights for each first bounce are recorded and the coefficient of restitution calculated for the empty ball (above) and for the same ball with added paper. The coefficient of restitution, calculated as the square root of the ratio of heights, reduces as the mass of loose paper is increased. A linear fit is a good approximation to preliminary data.

Fig 4 – Cr for a table-tennis ball versus the mass of tissue paper inside.
An alternative methods

i It can be shown that $C_r$ is given by the ratio of successive times of flight.

$$C_r = \frac{\Delta t_{n+1}}{\Delta t_n} \quad \text{…………. [2]}$$

Allowing objects to bounce on tiles and recording the sound, clearly defines the impact times and allows $C_r$ to be found in this way.

![Figure 3](image3.png)

**Fig 3** – recorded sound for a metal pipe bouncing vertically on tiles.

ii $C_r$ is by definition the ratio of relative velocities after and before impact. Finding the gradients of curve fits in a plot like figure 1 just before and just after impact determines $C_r$ without the effects of air resistance.

2 Playdough balls

Playdough, a mixture of flour, salt, and water, is more resilient than clay.

![Figure 4](image4.png)

**Fig 4** – the coefficient of restitution of a playdough ball bouncing on tiles.

$C_r$ for playdough balls on tiles reduces with increasing impact energy. The balls behave in a similar way to rubber balls.
Questions remain

Prove that $C_r$ (the ratio of relative velocities before and after collision) equals 1 for any elastic two-body collision that conserves momentum and kinetic energy. Prove also that equation 2 holds for any ball bouncing vertically on a level floor if air resistance can be neglected.

Explore these relationships

1. How does the coefficient of restitution versus impact energy vary for a range of rubber and/or plastic balls (solid or hollow) on a hard surface.

2. What is the temperature dependence of the coefficient of restitution for balls of a range of materials and construction, including table-tennis and squash balls of different qualities?

3. How does $C_r$ for table-tennis balls, squash balls and/or racquet balls, vary with and without a small hole, as a function of temperature?

4. How does the coefficient of restitution of selected hollow balls (table-tennis ball, practice golf ball etc.) vary with the addition of a known mass of loose paper, dry sand, or other materials.

5. Use an acoustic method to compare the coefficients of restitution for bars and pipes of different metals bouncing vertically on tiles.

Explore these additional relationships

1. How does $C_r$ vary for different compositions and temperatures for play dough? Use either a bed and glass balls or play dough balls in tiles.

2. What is the percentage contribution (if any) made to the temperature dependence of $C_r$ for table-tennis balls and/or squash balls by internal pressure?

3. Under what conditions (if any) does $C_r$ for balls of glass, rubber etc. depend on the radius of the bouncing ball?

4. Polymer balls can be made with borax, PVA wood glue, and flour. Make small balls with different flours, borax and PVA. Investigate their properties.

   Search the web for “borax ball” or see ...

   https://sciencebob.com/make-your-own-bouncy-ball/
5 Make ‘slime’ by mixing a spoon of PVA glue with 1-2 spoons of water and 3-4 spoons of talcum powder. Add a spoon of detergent and a drop of food colouring and more talc if required. Work well with a little baby oil. Place in a container overnight. As the mixture dries it takes on a rubbery texture.

Figure 5 shows the coefficient of restitution of the glue and talcum-powder ball shown as a function of temperature when dropped from 60 cm onto a hard surface (the base of the clamp stand).

Fig 5 – Cr as a function of temperature for a glue and talcum-powder ball.

The composite material behaves like a norborne with an increasing coefficient of restitution as temperature drops.

Suggestions

Experiment with balls made from mixtures of PVA, talcum-powder and detergent. Change the ratios of the ingredients and the water content to improve the bounce at room temperature.

Plot Cr versus temperature curves for your balls. Because the balls will absorb water, look for controlled environments at different temperatures: a refrigerator, an air conditioned room, a car parked in and out of direct sun, etc.

Plot Cr against ball radius and against the energy of impact. Compare with rubber balls of similar coefficient for a half-meter drop.
6 Make ‘slime’ as in 5 above, or some variation. Allow it to dry to a firm consistency over a week or so in an almost closed container. Make balls as above and make a bed of the material in a plastic container.

![Image of balls molded in measuring spoons.]

**Fig 6** – balls molded in measuring spoons.

**Suggestions**

*Measure and compare the coefficients of restitution of solid balls, (plastic, glass, steel, lead) when dropped on the bed of composite material with the restitution of “slime” balls of the same consistency when dropped on tiles as a function of temperature.*

*If Cr can be measured reliably by dropping glass balls on a flat bed the method is more convenient for a study of natural rubber and composite materials made with the freshly collected rubber. There may be interesting variations with the age and condition of the trees with different additives, and with different chemical treatments.*

7 It is said that Cr for steel balls dropped onto the freshly cut stump of an Australian Blue Gum is remarkably high.

**Suggestions**

*Compare the coefficients of restitution for steel balls when dropped onto a selected local wood with different grain directions and with different sap and water content.*